HW1 due 27/10/2021

(+2 points if you solve the problems with Julia)

Q.1

Root finding.

- There are many methods to find the roots of a function: e.g.
  - just plot it.
  - bisection.
  - iterative method: Newton's, secant, etc.

  Try these to solve for the **positive** root of $f(x) = x^2 - x - 1$. Of course the quadratic formula tells us the solution is

  $$x = \frac{1 \pm \sqrt{5}}{2}.$$

  (take the + root.) This is the Golden Mean.

- Show that the Golden Mean x satisfies

$$x = 1 + 1/x.$$

This provides yet another numerical scheme to solve for it, i.e. a recursive formula.

$$\begin{aligned} x_1 &\to 1 \\ x_2 &\to 1 + 1/x_1 \\ x_3 &\to 1 + 1/x_2 \\ \ldots \end{aligned}$$

Write a recursive function to implement this scheme to find the Golden Mean. (Iterate backward from N to 1) Write another one to get a factorial function.

Q.2

Babylonian Square Root.

- Use the following iterative scheme to find the square root of a:

$$x_{n+1} = \frac{1}{2}\,(x_n + \frac{a}{x_n}).$$

Starts with, e.g. $x_n = 100.0$ for $a = 49.0$, or any numbers to your liking.

- Show that this scheme is just the good old Newton's method.

Q.3

Finding inverses.

- Suppose we want to find the multiplicative inverse of a real number a. The answer is, of course, $1/a$. A fancy way to put it is that we want to solve for x such that

$$f(x) = 1/x - a = 0.$$

This becomes a root finding problem. Shows that the Newton's method gives

$$x_{n+1} = x_n\,(2 - a\,x_n).$$

Plug in numbers to verify that it is true.

- Show that the formula works also for finding an inverse of a matrix A.

$$X_{n+1} = X_n\,(2\,I - A\,X_n).$$

Demonstrate this with a 2x2 matrix.

Q.4

- Write a program to compute the Gaussian integral:

$$\int_{-\infty}^{\infty} dx\, e^{-A\,x^2}.$$

Check with the exact result

$$\sqrt{\frac{\pi}{A}}.$$

- If we define

$$\langle \hat{O}p \rangle = \frac{\int_{-\infty}^{\infty} dx\, e^{-(A/2)\,x^2}\, Op(x)\, e^{-(A/2)\,x^2}}{\int_{-\infty}^{\infty} dx\, e^{-A\,x^2}}.$$

Compute (numerically and analytically)

$$\begin{aligned}
&\langle x \rangle \\
&\langle x^2 \rangle \\
&\langle p \rangle = \langle -i\frac{d}{dx} \rangle \\
&\langle p^2 \rangle = \langle -\frac{d^2}{dx^2} \rangle.
\end{aligned}$$

Show that

$$(\langle p^2\rangle - \langle p\rangle^2) \times (\langle x^2\rangle - \langle x\rangle^2) = \frac{1}{4}.$$

Does it smell like quantum mechanics?

Q.5

Write a program to produce the bifurcation diagram of the logistic map:

$$x_{n+1} = r x_n \,(1 - x_n).$$

- for a given r within $(0, 4)$, iterate (starting from any initial value within $(0, 1)$) and collect the end point(s).
- plot the end points as a function of r to produce the bifurcation diagram.

Q.6

Visit the PDG website link to look up particle properties.

- Verify that the proton lives forever. (by forever, I mean it outlasts the age of the universe!)
- What about the neutron? What is its lifetime? What is the major decay mode? What is the fundamental interaction involved?
- How to define the N-baryons and the Δ-baryons?
- Look up the mass and width of $\Delta(1232)$ resonance. The major decay mode is into a pion and a nucleon, explain why the decay products should be in a relative P-wave, i.e. the orbital angular momentum $\ell = 1$? (instead of, e.g. S-wave)